MA'RUZA

ANIQ INTEGRALNI GEOMETRIK VA BOSHQA MASALALARNI YECHISHGA TATBIQI

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Tayanch so'z va iboralar: *egri chiziqli trapesiya, kesmada integral musbat va manfiy, funksiyani parametrik shakli, Dekart koordinalari, qutb koordinatalari, yoy uzunlgi, markaziy burchak, doiraviy sektor, siniq chiziq, integral yig'indi, o'tish formulalari, parametrik shakldagi tenglama. Perpendikulyar tekislik, qismiy oraliq, kesim konturi, silindrik jism, aylanma jism, moddiy nuqta, bajargan ish, inersiya momenti, koordinata bosimiga nisbatan inersiya momenti, sirt zichligi, sterjen, bir jinsli doira, kuchning yo'nalishi, Guk qonuni.*

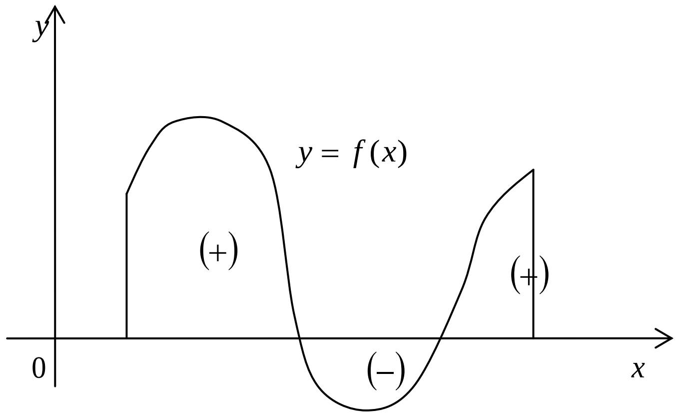
1. Figuralar yuzalarini Dekart va qutb koordinatalarida hisoblash

1. a) Bizga ma'lumki, agar $[a, b]$ kesmada uzlusiz bo'lgan $f(x) \geq 0$ bo'lsa, u holda $y = f(x)$ egri chiziq OX o'qi va $x = a$ hamda $x = b$ to'g'ri chiziqlar Bilan chegaralangan egri chiziqli trapesiyaning yuzi

$$S = \int_a^b f(x)dx \qquad (1)$$

bilan hisoblanar edi. Agar $[a, b]$ kesmada $f(x) \leq 0$ bo'lsa, u holda aniq integral $\int_a^b f(x)dx \leq 0$ bo'ladi. Absolyut qiymatiga ko'ra bu integralning qiymati ham tegishli egri chiziqli trapesiyaning yuziga teng:

$$S = \int_a^b |f(x)|dx \qquad (1')$$



1-shakl

Agar $f(x)$ funksiya $[a, b]$ kesmada ishorasini chekli son marta o'zgartirsa, u holda integralni butun $[a, b]$ kesmada qismiy kesmachalar bo'yicha integrallar yig'indisiga ajaratamiz. $f(x) > 0$ Bo'lgan kesmalarda integral musbat, $f(x) < 0$ bo'lgan kesmalarda integral manfiy bo'ladi. Butun kesmalar bo'yich olingan OX o'qidan yuqorida va pastda yotuvchi yuzalarning tegishli algebraik yig'indsini beradi. (2-shakl). Yuzalar yig'indisini odatdagi ma'noda hosil qilish uchun yuqorida ko'rsatilgan kesmalar bo'yicha olingan integrallar absalyut qiymatlari yig'indisini topish yoki $S = \int_a^b |f(x)|dx$ integralni hisoblanadi.

b) Agar $y_1 = f_1(x)$ va $y_2 = f_2(x)$ egri chiziqlar hamda $x = a$ va $x = b$ to'g'ri chiziqlar bilan chegarlangan figurani yuzini hisoblash kerak bo'lsa, u holda $f_1(x) \ge f_2(x)$ shart bajarilgan figuraning yuzi quyidagiga teng:

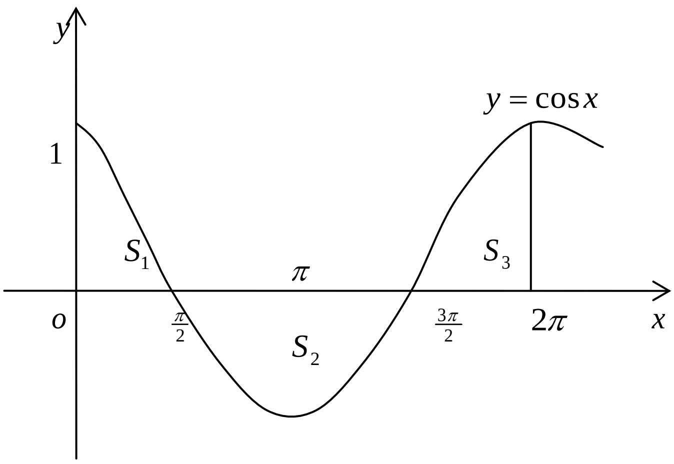
$$S = \int_a^b \left(f_1(x) - f_2(x)\right)dx \qquad (2)$$

1-misol. $y = \cos x,\ y = 0$ chiziqlar bilan chegaralangan yuzani $x \in [0, 2\pi]$ oraliqda hisoblang.

Yechish: Shaklini yasaymiz.

Formulga asosan $x \in \left[0, \frac{\pi}{2}\right]$ va $x \in \left[\frac{3\pi}{2}, 2\pi\right]$ da $\cos x \ge 0$ hamda

$x \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ da $\cos x \le 0$ bo'lagni uchun



2-shakl.

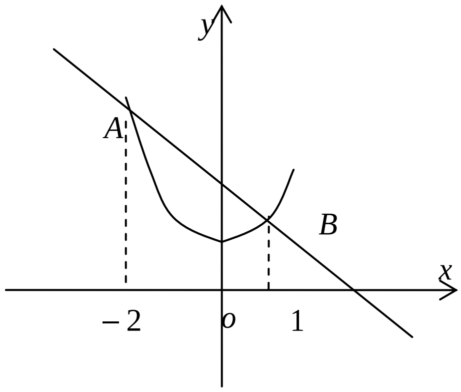
$$S = \int_0^{2\pi} |\cos x| dx = \int_0^{\frac{\pi}{2}} \cos x dx + \left| \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (-\cos x) dx \right| + \int_{\frac{3\pi}{2}}^{2\pi} \cos x dx = \sin x \Big|_0^{\frac{\pi}{2}} + |\sin x| \Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}} + \sin x \Big|_{\frac{3\pi}{2}}^{2\pi} =$$

$$= \sin\frac{\pi}{2} - \sin 0 + \left|\sin\frac{3\pi}{2} - \sin\frac{\pi}{2}\right| + \sin 2\pi - \sin\frac{3\pi}{2} = 1 + |-1-1| - (-1) = 4 \text{ kv.b.}$$

Demak, $S = 4$ kv.b.

2-misol. $y = x^2 + 1$ va $y = 3x$ chiziqlar bilan chegaralanagan figuraning yuzi hisoblansin.

Yechish: Figurani yasash uchun avval, ushbu sistemani $\begin{cases} y = x^2 + 1 \\ y = 3 - x \end{cases}$ yechib, chiziqlarni kesishish nuqtalarini topamiz.



Bu chiziqlar $A(-2,\ 5)$ va $B(1,\ 2)$ nuqtalarda kesishadi. U holda (2) formulaga asosan

$$S=\int_{-2}^{1}(3-x)dx-\int_{-2}^{1}(x^2+1)dx=\int_{-2}^{1}(2-x-x^2)dx=(2x-\frac{x^2}{2}-\frac{x^3}{3})\Big|_{-2}^{1}=(2-\frac{1}{2}-\frac{1}{3})-(-4-\frac{4}{2}-\frac{8}{3})=\frac{9}{2}=4{,}5$$

kv.b.

g) Agar egri chiziqli trapesiya hosil qiluvchi chiziqlar parametrik shaklidagi $x=\varphi(t),\ y=\psi(t)$ tenglamalari bilan berilgan bo'lsa, bunda bu tenglamalar $[a,\ b]$ kesmadagi biror $y=f(x)$ funksiyani aniqlaydi, bunda $t=\in[\alpha,\ \beta]$ va $\varphi(\alpha)=a,\ \psi(\beta)=b$. U holda egri chiziqli trapesiyani yuzini $S=\int_a^b ydx$ formula bilan hisoblash mumkin bo'ladi.

Bu integralda o'zgaruvchini almashtramiz $x=\varphi(t)$, $dx=\varphi'(t)dt$, $y=f(x)=f(\varphi(t))=\psi(t)$ bo'lganligidan $S=\int_\alpha^\beta \psi(t)\varphi'(t)dt$.

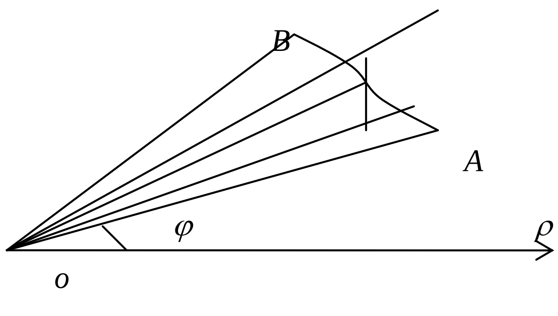
Bu formula chiziq parmetrik shakldagi tenglamasi bilan berilganda egri chiziqli trapesiyani yuzini hisoblash formulasidir.

3-misol. $x=a\cos t,\ y=b\sin t$ ellips bilan chegaralangan sohaning yuzi hisoblansin.

Yechish. Ellipsning yuqori yarim yuzini hisoblab, uni 2ga ko'paytirmiz $-a\le x\le a$ uchun $-a=a\cos t,\ \cos t=-1,\ t=\pi$. $a=a\cos t,\ \cos t=1,\ t=0$ ni topamiz, u holda formulaga asosan,

$$S=2\int_\pi^0 b\sin t(-a\sin tdt)=-2ab\int\sin^2 tdt=\pi ab \quad \text{kv.b.}$$

2. AB egri chiziq qutb koordinatalarida $\rho=\rho(\varphi)$ formula bilan berilgan va $\rho(\varphi)$ funksiya $[\alpha,\ \beta]$ kesmada uzluksiz bo'lsin.



Ushbu $\rho=\rho(\varphi)$ egri chiziq va qutb o'qlari bilan α va β burchak hosil qiluvchi 2ta $\varphi=\alpha,\ \varphi=\beta$ nurlar bilan chegaralanagan egri chiziqli sektorni yuzini hisoblaymiz. Buning uchun berilgan yuzani $\alpha=\varphi_0,\varphi_1,...,\varphi_n=\beta$ nurlar bilan n-ta ixtiyoriy qismlarga bo'lamiz. O'tkazilgan nurlar orasidagi burchaklarni $\Delta\varphi_1,\Delta\varphi_2,...,\Delta\varphi_n$ bilan belgilaymiz. φ_{i+1} Bilan φ_i orasidagi biror $\overline{\varphi}_i$ burchakka mos nurning uzunlgini $\overline{\rho}_i$ orqal belgilaymiz. Radiusi $\overline{\rho}_i$ va markaziy burchak

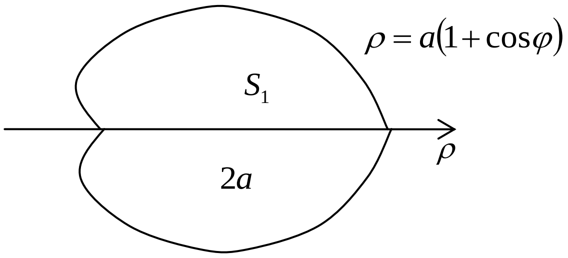
$\Delta\varphi_i$ bo'lgan doiraviy sektorni qaraymiz. Uning yuzi $\Delta S_i = \frac{1}{2}\bar{\rho}_i^2\Delta\varphi_i$ ga teng bo'ladi. U holda ushbu yig'indi

$S_n = \sum_{i=1}^{n}\bar{\rho}_i^2\Delta\varphi_i = \frac{1}{2}\sum_{i=1}^{n}\left[f(\bar{\rho}_i\right]^2\Delta\varphi_i$ Zinapoyasimon sektorni yuzini beradi. Bu yig'indi $\alpha < \varphi < \beta$ kesmada $\rho^2 = \left[f(\varphi)\right]^2$ funksiyaning integral yig'indisi bo'lganlgi sababali, uning limiti $\max\Delta\varphi_i \to 0$ da $\frac{1}{2}\int_{\alpha}^{\beta}\rho^2 d\varphi$ aniq integralga teng. Bu $\Delta\varphi_i$ burchak ichidagi qanday ρ_i nur olishimizga bog'liq emas. Demak, OAB sektorning yuzi $S = \frac{1}{2}\int_{\alpha}^{\beta}\rho^2 d\varphi = \frac{1}{2}\int_{\alpha}^{\beta}\left[f(\varphi)\right]^2 d\varphi$ formula bilan topilar ekan.

4-misol. $\rho = a(1+\cos\varphi),\ a > 0$ kardoida bilan chegarangan figuraning yuzini hisoblang.

Yechish: Kardioridani shaklini yasaymiz.

Formulaga asosan $S = 2S_1 = 2\frac{1}{2}\int_{\alpha}^{\beta}\rho^2 d\varphi = \int_{\alpha}^{\beta}\rho^2 d\varphi$



$$S = \int_0^{\pi} a^2(1+\cos\varphi)^2 d\varphi = a^2\int_0^{\pi}(1+2\cos\varphi+\cos^2\varphi)d\varphi = a^2\int_0^{\pi}(\tfrac{3}{2}+2\cos\varphi+\tfrac{1}{2}\cos 2\varphi)d\varphi =$$

$= a(\frac{3}{2}\varphi + 2\sin\varphi + \frac{1}{4}\sin 4\varphi)\Big|_0^{\pi} = = \frac{3}{2}\pi a^2$ Demak, karoidaning yuzi $S = \frac{3}{2}\pi a^2$ kv.b.

2. Egri chiziq yoyining uzunligini Dekart va qutb koordinatalarida hisoblash

a) Dekart koordinatalari sistemasida egri chiziq yoyining uzunligini hisoblash. Tekslikda to'g'ri burchakli koordinatalar sistemasida egri chiziq $y = f(x)$ tenglama berilgan bo'lsin. Bu egri chiziqning $x = a$ va $x = b$ vertekal to'g'ri chiziqlar orasidagi AB yoyning uzunligini hisoblaymiz. AB Yoyida abssissalari $a = x_0, x_1, ..., x_i, ..., x_n = b$ bo'lgan $A, M_1, M_2, ..., M_i, ..., B$ nuqtalarni olamiz va $AM_1, M_1M_2, ..., M_{n-1}B$ vatarlarni o'tkazamiz. Ularning uzunliklarini mos ravishda $\Delta S_1, \Delta S_2, ..., \Delta S_n$ lar bilan belgilaymiz. AB yoy ichiga chizilgan siniq chiziqning uzunligi $S_n = \sum_{i=1}^{n}\Delta S_i$ bo'lgani uchun AB yoyning uzunligi $S = \lim_{\max\Delta S_i \to 0}\sum_{i=1}^{n}\Delta S_i$ bo'ladi. Faraz qilaylik, $f(x)$ funksiya va uning $f'(x)$ hosilasi $[a, b]$ kesmada uzlusiz bo'lsin.

U holda $\Delta S_i = \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2} = \sqrt{1+\left(\frac{\Delta y_i}{\Delta x_i}\right)}\Delta x_i$ yoki Lagranj teoremasiga asosan $\frac{\Delta y_i}{\Delta x_i} = \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}} = f'(\xi)$ bo'lganligidan $(x_{i-1} < \xi_i < x_i)$ shartida $\Delta S_i = \sqrt{1+f'(\xi_i)^2}\Delta x_i$ bo'ladi. Ichki chizilgan siniq chiziqlarning uzunligi esa $S_n = \sum_{i=1}^{n}\sqrt{1+f'(\xi_i)^2}\Delta x_i$ bo'ladi. Shartga ko'ra, $f'(x)$ funksiya uzluksizdir, demak, $\sqrt{1+f'(\xi_i)^2}\Delta x_i$ funksiya ham uzluksizdir. Shuning uchun integral yig'indining limiti mavjud va u quyidagi aniq inetgralga teng:

$$S=\lim_{\max \Delta x_i\to 0}\sum_{i=1}^{n}\sqrt{1+f'(\xi_i)^2}\Delta x=\int_a^b\sqrt{1+f'(x)^2}dx$$

Demak, yoy uzunligini hisoblash formulasi $S=\int_a^b\sqrt{1+f'(x)^2}dx=\int_a^b\sqrt{1+(\frac{dy}{dx})^2}dx$ ko'rinishga ega bo'ladi.

Agar egri chiziq parametrik shaklidagi $x=\varphi(t), y=\psi(t)$ $(\alpha\le t\le\beta))$ tenglamasi bilan berilgan bo'lsa va $\varphi'(t)\ne 0$ bo'lganda bu tenglama biror $y=f(x)$ funksiyani aniqlaydi, bu funksiya uzluksiz bo'lib, $\frac{dy}{dx}=\frac{\psi'(t)}{\varphi'(t)}$ uzluksiz hosilaga ega, $a=\varphi(\alpha), b=\psi(\beta)$ bo'lsin. (9) integralda

$x=\varphi(t), dx=\varphi'(t)dt$ almashtirish bajaramiz. U holda

$$S=\int_\alpha^\beta\sqrt{1+(\tfrac{\varphi'(t)}{\psi'(t)})^2}\varphi'(t)dt \text{ yoki } S=\int_a^b\sqrt{(\varphi'(t))^2+(\psi'(t))^2}dt \text{ bo'ladi.}$$

Bu formulaga tenglamasi parametrik shaklda berilgan yoy uzunligini hisoblash formulasi deyiladi.

Qutb koordinatalar sistemasida egri chiziq yoyining uzunligini hisoblash.

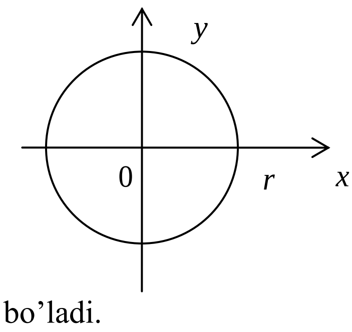
Tenglamasi qutb koordinatalar sistemasida bo'lgan $\rho=\rho(\varphi)$ egri chiziq berilgan bo'lsin. Qutb koordinatalaridan Dekart koordinatalariga o'tish formulasi $x=\rho\cos\varphi,\ y=\rho\sin\varphi$ dan foydalansak va unga formulani tatbiq qilsak $\frac{dx}{d\varphi}=\rho'\cos\varphi-\rho\sin\varphi$, $\frac{dy}{d\varphi}=\rho'\sin\varphi+\rho\cos\varphi$ bo'ladi. U holda $\left(\frac{dx}{d\varphi}\right)^2+\left(\frac{dy}{d\varphi}\right)^2=\left(f'(\varphi))^2+(f(\varphi)\right)^2=\rho'^2+\rho^2$ bo'ladi.

Demak, formulani ko'rinishi $S=\int_{\varphi_1}^{\varphi_2}\sqrt{\rho'^2+\rho^2}d\rho$ bo'ladi. Bu formulaga qutb koordinatalarida egri chiziq yoyining hisoblash formulasi deyiladi.

5-misol. $x^2+y^2=r^2$ aylana uzunlgi hisoblansin.

Yechish: Dastlab, aylananing birinchi kvadrantda yotgan qismini hisoblab, uni 4ga ko'paytiramiz. U holda AB yoy tenglamasi $y=\sqrt{r^2-x^2}$, $\frac{dy}{dx}=-\frac{x}{\sqrt{r^2-x^2}}$ bo'lganidan

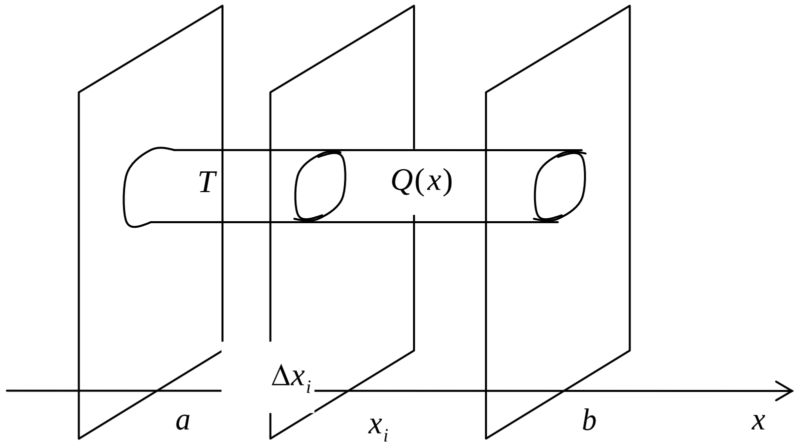
$$\frac{1}{4}S=\int_0^r\sqrt{1+\frac{x^2}{r^2-x^2}}dx=\int_0^r\frac{r}{\sqrt{r^2-x^2}}dx= r\arcsin\frac{x}{2}\bigg|_0^r=r\cdot\frac{\pi}{2}.$$



Butun aylana uzunligi $S=2\pi r$ bo'ladi.

3. Jism hajmini parallel kesimlarning yuzalari bo'yicha hisoblash

Biror T jism berilgan bo'lsin. Bu jismni OX o'qqa perpendikulyar tekislik bilan kesishdan hosil bo'lgan har qanday kesimni yuzi ma'lum deb faraz qilamiz. Bu holda yuza kesuvchi tekislikning vaziyatiga bog'liq, ya'ni x ning funksiyasi bo'lsin $Q(x)$.



$Q(x)$ ning $[a, b]$ oraliqda uzluksiz funksiya deb qarab, berilgan jism hajmini aniqlaymiz. Shu maqsadda $[a, b]$ oraliqda $x = x_0 = a, x = x_1, x = x_2, ..., x = x_n = b$ tekisliklarni o'tkazamiz. Har bir $x_{i-1} \le x \le x_i$ qismiy oraliqda ixtiyoriy ξ_i nuqta tanlab olamiz va i ning har bir qiymati uchun yasovchi x lar o'qiga parallel bo'lib, yo'naltruvchisi T jismni $x = \xi_i$ tekislik bilan kesishdan hosil bo'lgan kesimning konturidan iborat bo'lgan silindrik jism yasaymiz. Asosining yuzi $Q(x)$ ga, balandligi Δx_i bo'lgan bunday elementar silindrning hajmi $Q(\xi_i)\Delta x_i$ ga teng. Hamma silindrning hajmi $V_n = \sum_{i=1}^{n} Q(\xi_i)\Delta x_i$ bo'ladi. Bu yig'indidan $\max \Delta x_i \to 0$ dagi limitni hisoblasak, bu limit berilgan jismning hajmiga teng bo'ladi. $V = \lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} Q(\xi_i)\Delta x_i$. V_n miqdor $[a, b]$ kesmada uzluksiz $Q(x)$ fuknsiyaning integral yig'indisidir, shuning uchun bu limit mavjud va u, $V = \int_a^b Q(x)dx$ aniq integral bilan hisoblanadi.

1-misol. $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ ellipsoidning hajmi hisoblansin.

Yechish: Ellipsoidning OXZ teksilikka parallel bo'lib, undan x masofa uzoqlikdan o'tgan tekislik bilan kesganda yarim o'qlari $b_1 = b\sqrt{1 - \frac{x^2}{a^2}}$, $c_1 = c\sqrt{1 - \frac{x^2}{a^2}}$ bo'lgan

$$\frac{y^2}{\left(b\sqrt{1 - \frac{x^2}{a^2}}\right)^2} + \frac{z^2}{\left(c\sqrt{1 - \frac{x^2}{a^2}}\right)^2} = 1$$

ellips hosil bo'ladi. Bu ellipsning yuzi $Q(x) = \pi b_1 c_1 = \pi bc\left(1 - \frac{x^2}{a^2}\right)$. U holda, ellipsoidning hajmi formulaga asosan $V = \pi bc\int_a^b \left(1 - \frac{x^2}{a^2}\right)dx = \pi bc\left(x - \frac{x^3}{3a^2}\right)\Bigg|_{-a}^{a} = \frac{4}{3}\pi abc$ kub b.g teng bo'ladi.

4. Aylanma jismning hajmi

$y = f(x)$ egri chiziq, OX o'q va $x = a,\ x = b$ to'g'ri chiziqlar bilan chegaralngan egri chiziqli trapesiyaning OX o'qi atrofida aylanishdan hosil bo'lgan jismni qaraylik. Bu jismni abssissalar o'qiga perpendikulyar tekislik bilan kesishdan hosil bo'lgan ixtiyoriy kesma doira bo'ladi. Uning yuzi $Q(x) = \pi y^2 = \pi\left(f(x)\right)^2$. Hajmni hisoblash umumiy formlani qo'llab, aylanma jismning hajmini hisoblash formulasi

$$V = \pi\int_a^b y^2 dx = \pi\int_a^b \left(f(x)\right)^2 dx \qquad (13)$$

ni hosil qilamiz. Xuddi shuningdek OY o'q atrofida aylanishdan hosil bo'lgan jism hajmi

$$V = \pi\int_c^d x^2 dy = \pi\int_c^d \left(f(y)\right)^2 dy \qquad (14)$$

topiladi.

2-misol. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ellipsning OX va OY o'qlari atrofida aylantirish natijasida hosil bo'lgan jismlarning hajmini hisoblang.

Yechish: Ellips tenglamasidan $y^2 = \frac{b^2}{a^2}(a^2 - x^2)$, $x^2 = \frac{a^2}{b^2}(b^2 - y^2)$ topamiz va formulalarni qo'llab

$$V = 2V_1 = 2\pi\int_0^a y^2 dx = 2\pi\frac{b^2}{a^2}\int_0^a (a^2 - x^2)dx = 2\pi\frac{b^2}{a^2}(ax - \frac{x^3}{3})\Big|_0^a = 2\pi\frac{b^2}{a^2}(a^3 - \frac{a^3}{3}) = \frac{4}{3}\pi ab^2 .$$

Demak, $V = \frac{4}{3}\pi ab^2$ (kub b.). Endi OY atrofida aylanishdan hosil bo'lgan jismni hajmini topamiz

$$V = 2V_1 = 2\pi\int_0^b x^2 dy = 2\pi\frac{a^2}{b^2}\int_0^b (b^2 - a^2)dy = 2\pi\frac{a^2}{b^2}(b^2 y - \frac{y^3}{3})\Big|_0^b = 2\pi\frac{a^2}{b^2}(b^3 - \frac{b^3}{3}) = \frac{4}{3}\pi a^2 b .$$

Demak, $V = \frac{4}{3}\pi a^2 b$ (kub b.).

5. Ishni va jismlarni inersiya momentini aniq integral yordamida hisoblash

Biror F kuch ta'siri ostida M moddiy nuqta OS to'g'ri chiziq bo'yicha harakat qilsin, bunda kuchning yo'nalshi harakat yo'nalshi bilan bir xil bo'lsin. M nuqta $S = a$ holatdan $S = b$ holatga ko'chganda F kuchning bajargan ishi topilsin.

1) Agar F kuch o'zgarmas bo'lsa, u holda A ish F kuch bilan o'tilgan yo'l uzunligi ko'paytmasi bilan ifodalanadi $A = F(b - a)$

2) F kuch moddiy nuqtaning olgan o'rniga qarab uzluksiz o'zgarsin, ya'ni $[a, b]$ kesmada $F(S)$ uzluksiz funksiyani ifodlasak, u holda $[a, b]$ kesmani uzunliklari $\Delta S_1, \Delta S_2, ..., \Delta S_n$ bo'lgan n -ta ixtiyoriy bo'lakka bo'lamiz va har bir $[S_{i-1}, S_i]$ qismiy kesmada ixtiyoriy ξ_i nuqta tanlab olamiz. $F(S)$ Kuchning ΔS_i yo'lida bajargan ishini $F(\xi_i)\Delta S_i$ ko'paytma bilan almashtiriamiz. Oxirgi ifoda ΔS_i yetarlicha kichik bo'lganda F kuchning ΔS_i yo'lida bajarilgan ishning taqribiy qiymatini beradi:

$$A \approx A_n = \sum_{i=1}^{n} F(\xi_i)\Delta S_i$$

Bu yig'indidan $\max\Delta S_i \to 0$ da limiti $F(S)$ kuchning $S = a$ nuqtadan $S = b$ nuqtgacha bo'lgan yo'lda bajargan ishini ifodalaydi va $A = \int_a^b F(S)dS$ formula bilan hisoblanadi.

3-misol. Agar prujina 1 N kuch ostida 1 sm cho'zilishi ma'lum bo'lsa. Uni 4 sm cho'zish uchun qancha ish bajarish kerak?

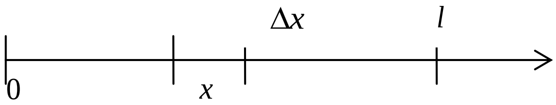
Yechish: Guk qonuniga ko'ra prujinani x m ga cho'zuvchi kuch $F = kx$ bilan topiladi. Agar $x = 0{,}01$ m va $F = 1$ N ekanlgini hisobga olsak, u holda $k = \frac{F}{x} = \frac{1}{0{,}01} = 100$ kelib chiqadi, bundan ga muvoffiq $A = \int_0^{0{,}04} 100x dx = 50x^2 \Big|_0^{0{,}04} = 0{,}08$ (J) ga teng bo'ladi.

3) XOY tekislikda massalari $m_1, m_2, ..., m_n$ bo'lgan $P_1(x_1, y_1), P(x_2, y_2), ..., P_n(x_n, y_n)$ moddiy nuqtalar sistemasi berilgan bo'lsin. Mexanikadan bizga ma'lumki, moddiy nuqtalar sistemasining O nuqtaga nisbatan inersiya momenti: $I_0 = \sum_{i=1}^{n}(x_i^2 + y_i^2)m_i = \sum_{i=1}^{n} r_i^2 m_i$ bunda $r_i = \sqrt{x_i^2 + y_i^2}$.

Faraz qilaylik, egri chiziq moddiy chiziqdan iborat bo'lib, u $y = f(x)$ tenglama bilan berilgan bo'lsin. Egri chiziqni chiziqli zichligi γ ga teng bo'lsin. Bu chiziqni uzunliklari $\Delta S_1, \Delta S_2, ..., \Delta S_n$ bo'lgan n ta bo'laklarga bo'lamiz, bunda $\Delta S_i = \sqrt{\Delta x_i^2 + \Delta y_i^2}$ ularning massalari $\Delta m_1 = \gamma \Delta S_1$, $\Delta m_2 = \gamma \Delta S_2$, ..., $\Delta m_n = \gamma \Delta S_n$ bo'lsin. Yoylarning har bir qismida abssissasi ξ_i va ordinatasi $\eta_i = f(\xi_i)$ bo'lgan nuqtalar olamiz. Yoyning O nuqtaga nisbatan inersiya momenti $I_0 \approx \sum_{i=1}^{n}(\xi_i^2 + \eta_i^2) \cdot \gamma \Delta S_i$. Agar $y = f(x)$ funksiya va uning hosilasi $f'(x)$ uzluksiz bo'lsa, u holda $\Delta x_i \to 0$ da yig'indi limitga ega va bu limit moddiy chiziqning inersiya momentini ifodalaydi:

$$I_0 = \gamma \int_a^b (x + f(x))\sqrt{1 + (f'(x))^2} dx$$

4) Uzunligi l bo'lgan ingichka bir jinsli tayoqchaning (sterjenning) oxirgi uchiga nisbatan inersiya momenti. Tayoqchani OX o'q kesmasi bilan ustma-ust joylashtiramiz, $0 \le x \le l$.

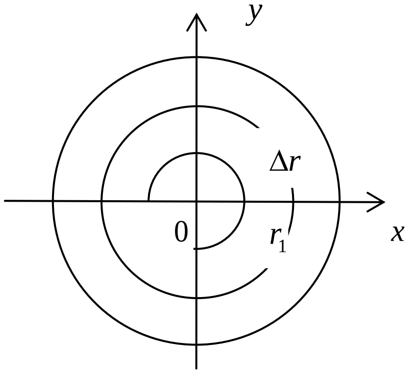


U holda $\Delta S_1 = \lambda x_1, \Delta S_2 = \gamma x_2, ..., r_i^2 = x_i^2$ bo'lib, formuladan $I_{ol} = \gamma \int_0^l x dx = \gamma \frac{l^3}{3}$ Agar tayoqchani massasi M berilgan bo'lsa, u holda $\gamma = \frac{M}{l}$ va formulaga ko'ra $I_{ol} = \frac{1}{3} M \cdot l^2$

5) Radiusi r bo'lgan aylananing markazga nisbatan inersiya momenti. Aylananing barcha nuqtalari uning markazidan bir xil masofada bo'lgan va massasi $m = 2\pi r \gamma$ uchun aylananning inersiya momenti

$I_0 = mr^2 = \gamma 2r \cdot r^2 = 2\pi r^3 \gamma$ bo'ladi.

6) Radiusi R bo'lgan bir jinsli doiraning markaziga nisbatan inersiya momentini topish uchun doirani n ta xalqalarga ajratamiz. S –doira yuzi birligini massasi bo'lsin. Bitta xalqani olib qaraymiz. Bu xalqning ichki radiusi r_i tashqi rdiusi $r_i + \Delta \kappa$ bo'lsin, massasi $\Delta m_i = \delta 2\pi r_1 \Delta r_1$ ga teng bo'ladi. Bu massani markazga nisbatan inersiya momenti formulaga asosan $(\Delta I_0) \approx \delta 2\pi r_i \Delta r_i \cdot r_i^2 = \delta 2\pi r_i^3 \Delta r_i$ ga teng. Bu doiraning inersiya momenti



$I_0 \approx \sum \delta 2\pi r_i^3 \Delta r_i$. Bundan $\Delta r_i \to 0$ da limitga o'tsak $I_0 = \delta 2\pi \int r^3 dr = \pi\delta \frac{R^2}{2}$ formulani hosil qilamiz. Agar doiraning massasi M bo'lgan bo'lsa, u holda sirt zichligi δ -quyidagiga teng $\delta = \frac{M}{\pi R^2}$. Buni ga qo'ysak, $I_0 = \frac{MR^2}{2}$ bo'ladi.